PROBLEMS for PRACTICE

SECTION – I

1. The resultant of two forces each of magnitude P acting at a point is $\sqrt{2}P$. What is the angle between the vectors? **(90°)**
2. Two intersecting vectors of equal magnitude have a resultant of magnitude one and a half times the magnitude of either vector. What is the angle between the vectors? **(82° 49')**
3. Find the angle between two vectors if their resultant is equal to either of them. **(120°)**
4. If the square of the resultant of two forces acting at a point is three times the product of the forces, at what angle do they act? **(60°)**
5. Sum of two forces is 8 kg wt. The magnitude of the resultant of the two forces is 4 kg wt and it is at right angles to the smaller force. Find the forces. **(5 kg wt; 3 kg wt)**
6. The maximum value of resultant of two forces is 5N and the minimum value is 1N. Find the resultant of the forces when they act at an angle of 60°. **(4N)**
7. The resultant of two forces acting at a point making an angle of 60° is $\sqrt{13}$N. When the two forces act at right angles to each other, the resultant is $\sqrt{10}$N. Find the magnitudes of the forces. **(3N; 1N)**
8. The sum of the magnitudes of two forces acting at a point is 18N and the magnitude of their resultant is 12N. If the resultant makes an angle of 90° with the force of smaller magnitude, find the magnitude of each force. **(5N; 13N)**

9. The resultant of two vectors 3P and 2P is R. If the first vector is doubled, the resultant vector is also doubled. Find the angle between the vectors. **(120°)**

10. The magnitude of the resultant of two intersecting forces P and Q is equal to the magnitude of P. Show that the resultant of forces 2P and Q acting at the same angle is perpendicular to Q.

11. For two equal non-parallel forces acting at a point, the square of the resultant is 3 times the product of the forces. What is the angle between the forces? **(60°)**

12. Two forces P and Q making an angle of 60° with each other act at a point. P has a magnitude of 8 N and the resultant of the two forces has a magnitude of $\sqrt{84}$ N. Find the magnitude of Q and the direction of the resultant. **(2 N; 49°6' with Q)**

13. Two forces of 5 N each acting at a point are at right angles. At what angle two forces 4 N each act at the same point to balance the first two? **(50° appx.)**

14. P and 2P are two forces acting on a particle. When the first force is increased by 20 kg wt and the second force is doubled, the direction of resultant force remains unchanged. Find P. **(20 kg wt)**

15. Two forces acting on a particle in opposite direction have a resultant of 10 N. Had they acted at right angles to each other, their resultant would have a magnitude of 50 N. Find the magnitude of forces. **(30 N; 40 N)**

16. The velocity of a body is 20 m/s making an angle of 30° with the horizontal. Find its horizontal and vertical components. **(17.32 m/s; 10 m/s)**

17. Find the components along X and Y axes of a vector 20 units long when it makes an angle of 30° with the X-axis. **($10\sqrt{3}$ units; 10 units)**

18. A cart kept on a horizontal plane exerts a force of 500 N on the plane. The cart is now pulled along the plane by applying a force of 100 N making an angle of 30° with the horizontal. Find the force pulling the cart forward. What force is experienced by the plane now? **(86.6 N, 450 N)**

19. Vector $\vec{A}$ has X and Y components 2 cm and 2 cm respectively. Vector $\vec{B}$ has X and Y components 2 cm and - 4 cm respectively. If $\vec{A} - \vec{B} - \vec{C} = 0$, what are the components of $\vec{C}$ along X and Y axes? What is the magnitude of $\vec{C}$? **(2 cm; 8 cm; $\sqrt{68}$)**

20. The X and Y components of a displacement vector are (15, 7) m. Find the magnitude and direction of $\vec{A}$ **(16.55 m; appx 65° with the Y-axis)**

21. The co-ordinates of a point P in a system of rectangular co-ordinates with origin at O are (1, 2, -3). Find $\vec{PQ}$. **($\hat{i} + 2\hat{j} - 3\hat{k}$)**

22. Points P and Q have co-ordinates (1, 2, 3) and (4, 5, 6). Find $\vec{PQ}$. **$3(\hat{i} + \hat{j} + \hat{k})$**

23. Find the distance between the points P (3, 4, -7) and Q (-2, 5, 10). **(17.75 units)**

24. Sometimes a vector is described in terms of its components by using parenthesis notation. $\vec{A}$ is written as $\vec{A} = (A_x, A_y, A_z)$. What angle does the vector $\vec{A} = (1, 1, 1)$ make with the X, Y, Z axes? **(54.73° with each axis)**

25. A body situated at (3, 2, 1) is given a displacement $3\hat{i} - 2\hat{j} + 2\hat{k}$. What are the co-ordinates of its new position? **(6, 0, 3)**

26. Two unit vectors act making an angle of (1) 30° (2) 60° with each other. What is the magnitude of their resultant? **(1.93 ; 1.732)**

27. If $\vec{P} = 3\hat{i} - \hat{j} + 6\hat{k}$, find P. **($\sqrt{46}$)**

28. If $\vec{A} = \hat{i}A_1 + \hat{j}A_2 + \hat{k}A_3$, find A. **$\left(\sqrt{A_1^2 + A_2^2 + A_3^2}\right)$**

29. Find the direction cosines of vector $\vec{A} = 2\hat{i} + 2\hat{j} + \hat{k}$. **$\left(\frac{2}{3}; \frac{2}{3}; \frac{1}{3}\right)$**

30. Vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are related by $\vec{A} + \vec{B} = \vec{C}$ and $A + B = C$. How is $\vec{A}$ oriented w.r.t. $\vec{B}$?
(same direction)

31. If $\vec{OA} = 2\hat{i} - 5\hat{j} + 7\hat{k}$ and $\vec{OB} = 5\hat{i} - 3\hat{j} + \hat{k}$, find $\vec{AB}$ and its magnitude. $(3\hat{i} + 2\,2\hat{j} - 6\hat{k}\,;\, 7)$

32. $\vec{R}$ is the resultant of vectors $\vec{P} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{Q} = \hat{i} + 2\hat{j} + \hat{k}$. Find the unit vector in the direction of $\vec{R}$. $(3\hat{i} + \hat{j} + 5\hat{k}) / \sqrt{35}$)

33. The resultant of two vectors $\vec{A} = 2\hat{i} + 3\hat{j} - 7\hat{k}$ and $\vec{B} = 5\hat{i} - 7\hat{j} + \hat{k}$ is a vector $\vec{C}$. Find $\vec{C}$ and its magnitude. $(7\hat{i} - 4\hat{j} - 6\hat{k} / \sqrt{101}$)

34. Find the unit vector parallel to $4\hat{i} - 3\hat{j} - 12\hat{k}$. $\left(\frac{4\hat{i} - 3\hat{j} - 12\hat{k}}{13}\right)$

35. What vector added to $\hat{i} - 2\hat{j} + 5\hat{k}$ and $2\hat{i} + \hat{j} - 3\hat{k}$ will give a unit vector along the Y-axis?
$(-3\hat{i} + 2\hat{j} - 2\hat{k})$

36. Show that the vectors $\vec{P} = 2\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{Q} = -6\hat{i} + 9\hat{j} + 3\hat{k}$ are parallel.

37. Find x if $\vec{A} = \hat{i} - 3\hat{j} + 4\hat{k}$ and $x\hat{i} - 6\hat{j} + 8\hat{k}$ are parallel. $(x = 2)$

38. Find $\vec{P} \cdot \vec{Q}$ where $\vec{P} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{Q} = \hat{i} - \hat{j} + 2\hat{k}$ (3)

39. Find the angle between the vectors $\vec{A} = 4\hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{B} = \hat{i} + 4\hat{j} + 3\hat{k}$. $(90°)$

40. If $\vec{P} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{Q} = 3\hat{i} + \hat{j} + 2\hat{k}$, find the angle between $\vec{P}$ and $\vec{Q}$ $(60°)$

41. If $\vec{P} = \hat{i} + 12\hat{j} + \hat{k}$ and $\vec{Q} = 2\hat{i} - a\hat{j} + 4\hat{k}$ are perpendicular, find a. (1/2)

42. Under what conditions will the vectors $\vec{P} = 3\hat{i} - 5\hat{j} + 5\hat{k}$ and $\vec{Q} = 5\hat{i} - \hat{j} + x\hat{k}$ be perpendicular to each other? $(x = -4)$

43. Vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ join the origin of a system of rectangular coordinates to points A, B and C respectively. Show that ABC is a right angled triangle.

44. Given $\vec{P} = \hat{i} - 2\hat{j} - 3\hat{k}$ and $\vec{Q} = 4\hat{i} - 2\hat{j} + 6\hat{k}$, find the angle made by $\vec{P} + \vec{Q}$ with the X-axis. $(45°)$

45. The angle between vectors $\vec{A}$ and $\vec{B}$ is θ. Find $\vec{A} \cdot (\vec{A} \times \vec{B})$. (Zero)

46. $\vec{A} = \hat{i}A_1 + \hat{j}A_2 + \hat{k}A_3$ and $\vec{B} = \hat{i}B_1 + \hat{j}B_2 + \hat{k}B_3$, find $\vec{A} \cdot \vec{B}$ and $\vec{A} \times \vec{B}$
$\{(A_1B_1 + A_2B_2 + A_3B_3);$
$[(A_2B_3 - A_3B_2)\hat{i} + (A_3B_1 - A_1B_3)\hat{j} + (A_1B_2 - A_2B_1)\hat{k}]\}$

47. Evaluate (a) $3\hat{i} \times 5\hat{j}$ (b) $2\hat{j} \times 3\hat{k}$ (c) $5\hat{k} \times 3\hat{i}$ (d) $\hat{k} \times 9\hat{j}$ (e) $\hat{i} \times (\hat{j} \times \hat{k})$ (f) $\hat{j} \times (\hat{j} \times \hat{i})$
[(a) $15\hat{k}$ (b) $6\hat{i}$ (c) $15\hat{j}$ (d) $-9\hat{i}$ (e) zero (f) $-\hat{i}$]

48. Find the area of the parallelogram formed by the vectors
$\vec{P} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{Q} = 2\hat{i} + 2\hat{j} - 5\hat{k}$. $(\sqrt{138}$ square units)

49. Find the area of the parallelogram formed with $\vec{A} = 3\hat{i} - 4\hat{j} + 2\hat{k}$ and $B = 2\hat{i} + 3\hat{j} + 4\hat{k}$ as the adjacent sides measured in metres. Hence find the area of the triangle formed by the two vectors as the adjacent sides $(28.93\ m^2;\ 14.465\ m^2)$